Statistical Modeling of the Interaction of Multiple Signals in Nonlinear RF Systems

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Abstract — A nonlinear statistical analysis for modeling distortion, cross modulation, and jammer susceptibility of CDMA signals in a nonlinear RF system is presented. The development of the statistical technique is presented here and applied to various interference scenarios. It is shown that relatively higher nonlinearities must be included in the RF system model so that nonlinear distortion is adequately captured.

I. INTRODUCTION

The nonlinear interaction of multiple signals, or cross modulation, is a fundamental phenomena in a number of related situations including co-site interference, multi-carrier systems, jamming, and multifunctional systems where multiple communication and radar channels are routed through the same RF hardware. For example, one of the stringent requirements in CDMA receiver design is the proper reception of a CDMA channel in the presence of a single tone jammer. In commercial environments, the single tone jammer models a narrow band AMPS signal transmitted from an AMPS base station or mobile unit. When both signals interact in a nonlinear receiver, the jammer, which is a problem when it is the largest signal, can reduce the sensitivity of the receiver to the CDMA signal. In addition, the intermodulation of the signal and jammer can lead to spurious CDMA-like signals that both affect our ability to extract the desired signal and can also result in spurious radiation of the generated signals. In a military situation, the jammer could be an intentional interferer.

Previously we developed a statistical technique which utilized the autocorrelation function and moments of the input signal to determine the statistics of the signal at the output of a nonlinear system [1], [2]. The present work extends this to the multiple channel case and results are presented for multiple carrier amplifier with multiple CDMA signals.

The major contribution of this paper is the development of an analytical formulation of the cross modulation problem in CDMA receiver where we give estimates of the interference introduced by the presence of a single tone jammer. A general autocorrelation function for the output signal is derived for the sum of two CDMA modulated carriers and for the sum of a modulated carrier and a single tone jammer passed through a memoryless nonlinear model.

II. CDMA SYSTEM MODEL AND STATISTICS

The nonlinear device can be characterized by using single tone measurements and can be represented as a memoryless bandpass nonlinearity with complex transfer characteristics where complex power series expansion is used to model the instantaneous AM-AM and AM-PM characteristics [1]:

$$G(z(t)) = = \sum_{n=0}^{N} a_n z^n(t) \qquad (1)$$

where a_n is a complex power series coefficient and $z(t)$ is the complex envelope of the digitally modulated CDMA signal. The CDMA signal model is given by:

$$w(t) = Re\{z(t)e^{j\omega_c t}\} = A(t)cos(\omega_c t + \theta(t)) \qquad (2)$$

where $z(t) = A(t)e^{j\theta(t)} = x(t)+jy(t)$ is the complex envelope of the modulated signal $w(t)$. The complex envelope $z(t)$ has a bandwidth of B which is the bandwidth of the PN code. The nonlinear gain characteristic is assumed to be bandpass nonlinearity containing no significant memory within the bandwidth of the modulation. Thus the AM-AM and AM-PM nonlinearities respond instantaneously to amplitude changes from the modulated carrier signal. The complex gain expression for the fundamental transfer characteristics is:

$$G(w(t)) = \sum_{n=0}^{N} a_n w^n(t) \qquad (3)$$

$$w^n(t) = \frac{1}{2^n} \sum_{k=0}^{n} \binom{n}{k} [z(t)]^k [z^*(t)]^{n-k} e^{j\omega_c(2k-n)t}$$

Considering only the terms centered at the carrier frequency, this implies that $2k - n = \pm 1$, then the complex envelope of (3) can be expressed as:

$$\tilde{G}(z(t)) = z(t) \sum_{n=0}^{N} \frac{a_n}{2^{n-1}} \binom{n}{\frac{n+1}{2}} [z(t)z^*(t)]^{\frac{n-1}{2}}$$

This expression describes the complex envelope of the first harmonic of a modulated carrier signal passed through a bandpass nonlinear circuit described by complex power series. The complex envelope of the output autocorrelation function of the nonlinear power series model is [1]:

$$\tilde{R}_{gg}(\tau) = \sum_{m=0}^{N} \sum_{n=0}^{N} \frac{a_n a_m^*}{2^{n-m-2}} \binom{n}{\frac{n+1}{2}} \binom{m}{\frac{m+1}{2}} R_{z_n z_m}(\tau) \quad (4)$$

where:

$$R_{z_n z_m}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} z_1^{\frac{n+1}{2}} z_1^{*\frac{n-1}{2}} z_2^{\frac{m-1}{2}} z_2^{*\frac{m+1}{2}} dt$$

assuming that $z(t)$ is an ergodic wide sense stationary process. The output power spectrum is obtained from the Fourier transform of the output autocorrelation function:

$$S_{gg}(f) = \sum_{m=0}^{N} \sum_{n=0}^{N} \frac{a_n a_m^*}{2^{n-m-1}} \binom{n}{\frac{n+1}{2}} \binom{m}{\frac{m+1}{2}} S_{nm}(f) \quad (5)$$

where $S_{nm}(f) = \int_{-\infty}^{\infty} R_{z_n z_m}(\tau) e^{-j\omega\tau} d\tau$.

Therefore, the output spectrum is a sum of the Fourier transform of each component of the autocorrelation function weighted by the appropriate power series coefficient.

III. CROSS MODULATION ANALYSIS

In the problem of cross modulation we are interested in analyzing the behavior of nonlinear power amplifier in the presence of the sum of two CDMA signals at its input. To establish the model of this effect let us consider an input $w(t)$ which consists of the sum of two modulated carriers at the input of the nonlinear amplifier $w_1(t)$ and $w_2(t)$, where $w(t) = w_1(t) + w_2(t)$ and $w_1(t)$ and $w_2(t)$ are modulated at frequencies ω_1 and ω_2 respectively. Let $z(t)$ and $v(t)$ be the complex envelopes of the modulated signals $w_1(t)$ and $w_2(t)$, then:

$$w(t) = w_1(t) + w_2(t)$$
$$= A_1(t) \cos(\omega_1 t + \theta_1) + A_2(t) \cos(\omega_2 t + \theta_2)$$
$$= \frac{1}{2} z(t) e^{j\omega_1 t} + \frac{1}{2} z^*(t) e^{-j\omega_1 t} + \frac{1}{2} u(t) e^{j\omega_2 t} + \frac{1}{2} u^*(t) e^{-j\omega_2 t}$$
$$= \frac{1}{2} \left(z(t) + u(t) e^{j(\omega_2 - \omega_1)t} \right) e^{j\omega_1 t}$$
$$+ \frac{1}{2} \left(z(t)^* + u^*(t) e^{-j(\omega_2 - \omega_1)t} \right) e^{-j\omega_2 t}$$

The complex envelope of the sum signal $w(t)$ is:

$$v(t) = z(t) + u(t) e^{j(\omega_2 - \omega_1)t} = z(t) + u(t) e^{j\Delta\omega t}$$

where $\Delta\omega = \omega_2 - \omega_1$.

The receiver desensitization problem can be treated as a special case of the above analysis. Let the input signal $w(t)$ consist of a single tone jammer $w_1(t) = A_1 \cos(\omega_1 t)$ and a transmitter leakage CDMA signal $w_2(t) = A_2(t) \cos(\omega_2 t + \theta_2)$, then the output autocorrelation can be expressed as:

$$\tilde{R}_{gg}(\tau) = \sum_{m=0}^{N} \sum_{n=0}^{N} \frac{a_n a_m^*}{2^{n-m-2}} \binom{n}{\frac{n+1}{2}} \binom{m}{\frac{m+1}{2}} R_{v_n v_m}(\tau) \quad (6)$$

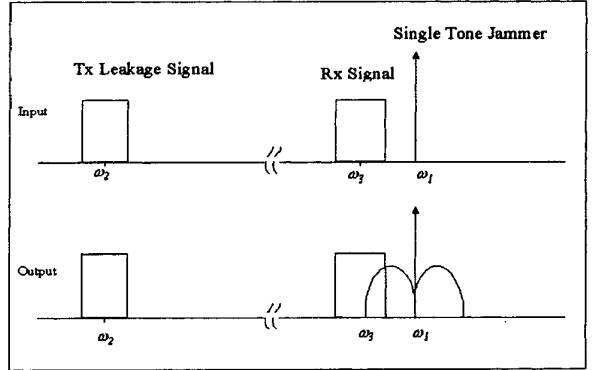where $z(t) = A_2(t) e^{j\theta(t)}$, $u(t) = A_1$.

Fig. 1 Cross modulation process.

To estimate the interference introduced inside the band of the received CDMA signal, let the received signal be centered at frequency ω_3, then the total power introduced by the jammer cross modulation that penetrates into receiver band as shown in Fig. 1 is:

$$P_{cross\_mod} = \int_{\omega_3 - \pi B}^{\omega_3 + \pi B} S_{gg}(\omega) d\omega \quad (7)$$

III. SIMULATION RESULTS AND DISCUSSION

The autocorrelation analysis in Section III was used to calculate the output autocorrelation of an integrated RF power amplifier for different kinds of excitation (2 CDMA signals and a CDMA signal and a single tone jammer). The amplifier is the GaAs MESFET 900 MHz driver amplifier described in [1], where a complex power series of order 13 was fitted to the AM-AM and AM-PM measured data. The estimation of the output autocorrelation function was done using a biased autocorrelation estimator. In turn, the output power spectra were calculated from the FFT of the autocorrelation time sequences. All simulations were done in MATLAB.

A. Single Tone Jamming

The cross modulation results derived above are applied to the particular case of single-tone jamming of a CDMA signal. This situation is described in the IS-98 standard [3], as representing the interference of an AMPS signal having 30 kHz bandwidth with CDMA channel having a 1.23 MHz bandwidth. The cross modulation interference was simulated with a single tone jammer at an offset of 1.23 MHz from the receiver carrier and a CDMA leakage signal and separated by 17.23 MHz at the input of the nonlinear amplifier. The cross power interference is measured as the total power inside the band of the received signal as shown in Fig 1. Fig. 2 (a) and (b) shows the output of the nonlinear amplifier driven by the sum of a CDMA leakage signal and a single tone jammer. The multiple curves show different power levels of the Tx leakage signal or jammer signal. It also shows the intermodulation products that appear because of the mixing property of the nonlinear device. Fig. 3 shows the interference introduced by the cross modulation inside the receiver band vs. (a) Tx leakage power with jammer power fixed at −20 dBm and vs. (b) vs. jammer power with Tx leakage power fixed at −20 dBm. The Tx leakage power and jammer input power were swept over a range of −35 to −5 dBm.

B. Multiple CDMA Channels

The case where the nonlinear amplifier is driven by the sum of two CDMA signals is also simulated using the formulation of Section III. Fig. 4 shows the output power spectra of the sum of two CDMA signals at an input power of −15 dBm and separated by 10 MHz. Fig. 5 shows the increase in ACPR due to cross modulation vs. output power.
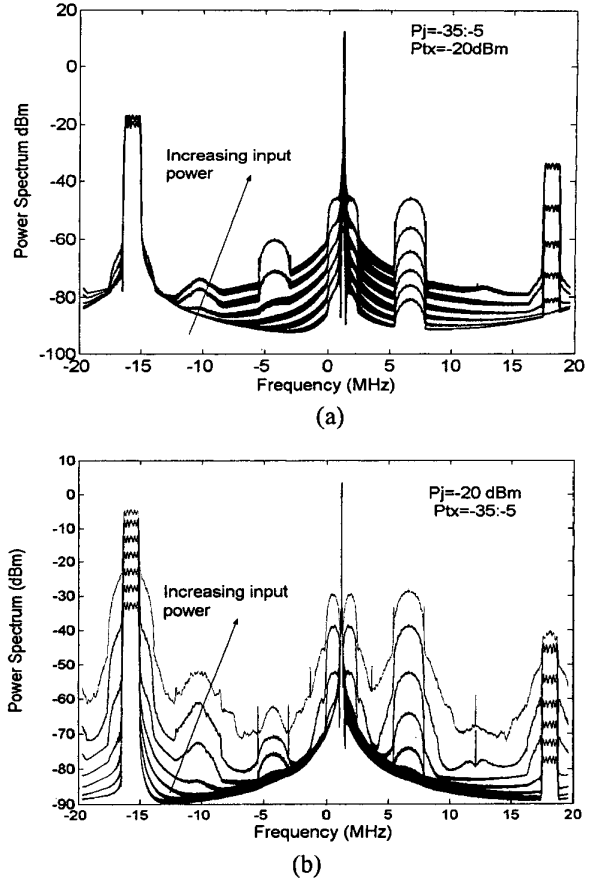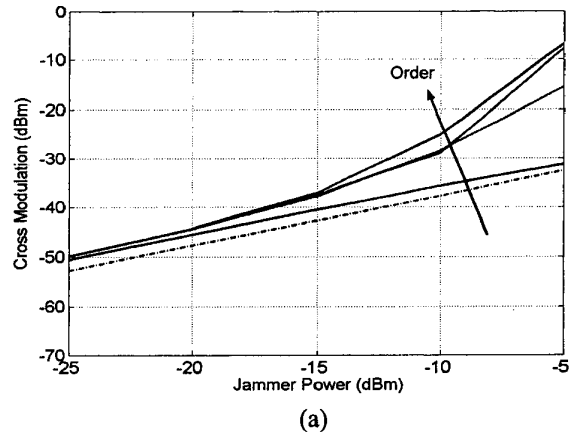

(a)

(b)

Fig. 2 Cross modulation and intermodulation products as a function of (a) Jammer power, (b) Tx leakage power. Multiple curves indicate different power levels.
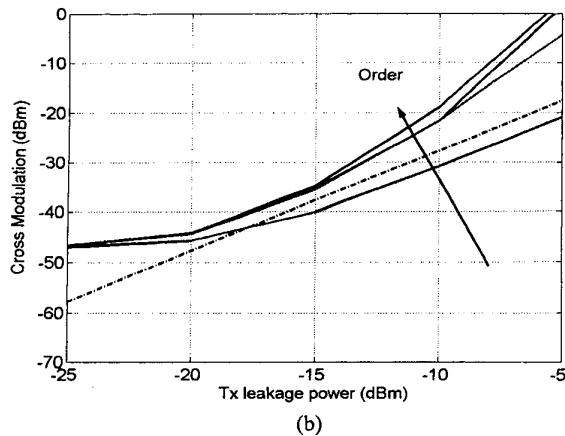

(a)

Fig. 3 Cross modulation vs. (a) Jammer power, (b) Tx leakage power (solid), using empirical formula [4] (dashed), multiple curves represent order nonlinearity (n=3, 5, 9, 13).
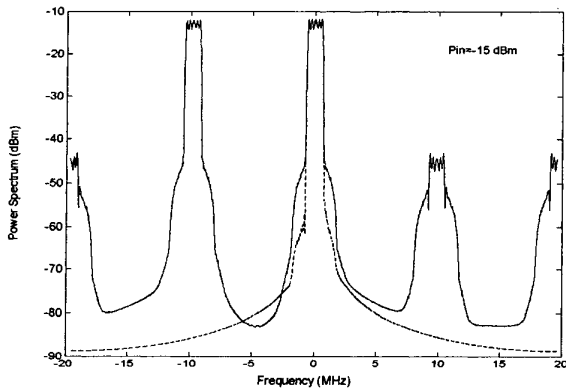

Fig. 4 Spectra of the output of nonlinear amplifier driven by a single input (dashed) and the sum of two CDMA signals separated by 10 MHz (solid).

V. CONCLUSION

The key result of this is the development of behavioral models of a nonlinear RF system. This behavioral model has two forms: one is a complex power series form that can be developed from single tone measurements and the second is a general statistical behavioral model that handles single and multiple channels. Most importantly, it is shown that the two models are related and in particular how the statistical behavioral model can be developed from the complex power series model. It was shown that the order of nonlinearity has a considerable effect on the accuracy of simulations, which renders the third order assumption

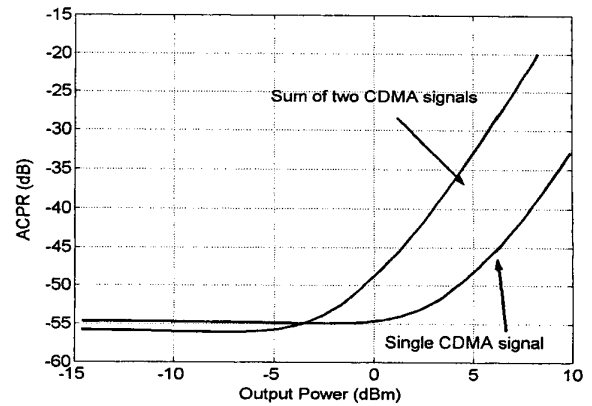for nonlinear amplifiers inaccurate in estimating the output power spectrum.

Fig. 5 The increase in ACPR when the amplifier is driven by the sum of two CDMA signal.

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